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| Name of Course | : CBCS B.Sc. Mathematical Sciences |
| Unique Paper Code | : 42354401_OC |
| Name of Paper | : Real Analysis |
| Semester | : IV |
| Duration | : 3 hours |
| Maximum Marks | : 75 Marks |

Attempt any four questions. All questions carry equal marks.

1. State and prove the condition for a sequence of real numbers to have a convergent subsequence. Is the converse true? Give example. Is every Cauchy sequence bounded? If yes, prove it.

2. Using the definition prove the convergence of the following sequences:

$$(i) \lim_{n \rightarrow \infty} \frac{\cos n\alpha}{n+1} = 0 \quad (ii) \lim_{n \rightarrow \infty} \frac{n^3-1}{3+2n^3} = \frac{1}{2} \quad (iii) \lim_{n \rightarrow \infty} \frac{1}{(n+1)^2+1} = 0$$

Further, find all $x \in \mathbb{R}$ that satisfy the inequality: $|x| + |x+1| < 2$

3. Check the convergence or divergence of the following series

$$(i) \sum_{n=1}^{\infty} \sqrt{n^3+1} - \sqrt{n^3-1} \quad (ii) \sum_{n=1}^{\infty} \frac{7^{n+1}}{9^n}$$

$$(iii) \sum_{n=1}^{\infty} \frac{n^3}{3^n} \quad (iv) \frac{1}{e} + \frac{4}{e^2} + \frac{27}{e^3} \dots$$

4. For each of the following, determine whether the series converges absolutely, converges conditionally, or diverges.

$$(i) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{4n+5} \quad (ii) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{n+5}$$

$$(iii) \sum_{n=1}^{\infty} \frac{\sin n\pi/2}{n^{5/2}} \quad (iv) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \log n}$$

5. Check pointwise and uniform convergence of the sequence $\langle f_n \rangle$ defined as $f_n = nxe^{-nx^2}$ on $[0,1]$. Show that the series $\sum_{n=1}^{\infty} \frac{\sin(x^2+nx^2)}{n(n+1)}$ is uniformly convergent for all real values of x . Further find the radius of convergence and interval of convergence for the power series

$$(i) \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)} \quad (ii) \sum_{n=0}^{\infty} \frac{n! (x+2)^n}{n^n}$$

6. Let f be a function on $[a, b]$ and P be a partition of $[a, b]$ and Q be its refinement, then show that

$$U[Q, f] - L[Q, f] \leq U[P, f] - L[P, f]$$

If f is defined on $[0, 1]$ by $f(x) = 2x^2 \forall x \in [0, 1]$, then prove that f is Riemann integrable. Also find the value of integral.

Further show that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, -1 \leq x \leq 1$ and hence deduce that

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \dots$$

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